Social Behavior and Information Diffusion

Representative Agent Hypothesis vs. Agent-based Modeling

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Outline

Outline

Background: RAH vs. ABM

What is the Representative Agent Hypothesis?

- The representative-agent hypothesis is a hypothesis in which all agents behave in such a way that their collective actions are assuming to be the actions of one agent making an optimal decision.
- The representative-agent hypothesis allows for greater ease in solution procedures.
  - It is easier to find the equilibrium (relatively...).
  - This is usually called the **analytical optimization** .

Background: RAH vs. ABM

What is the Representative Agent Hypothesis?

- **Examples of the representative-agent models:**
  - Profit maximization, utility maximization, or cost/loss minimization...
- **Methods of optimization:**
  - (1) First-order condition - unconstrained optimization
  - (2) Lagrangian multiplier - constrained optimization
  - (3) Dynamic optimization
    - (a) Bellman equation (over discrete time), and
    - (b) Hamiltonian multiplier (over continuous time).

Background: RAH vs. ABM

What is Agent-based Modeling?

- Agent-based modeling has been considered as a bottom-up approach modeling behaviors of a group of agents, rather than a representative agent, in a system.
- LeBaron and Tesfatsion (2008, 246): "Potentially important real-world factors such as subsistence needs, incomplete markets, imperfect competition, inside money, strategic behavioral interactions, and open-ended learning that tremendously complicate analytical formulations are typically not incorporated"

Background: RAH vs. ABM

What is Agent-based Modeling?

- One important element of ABM is that it allows the possibility of agents' interactions in micro levels with the assumption of bounded-rationality or imperfect information.
- Given agents' heterogenous characteristics and their interactions at the micro level, we can simulate the system and observe changes in the macro level over time according to the system-simulated data.

This Presentation

How can EITM be applied to the research of:

1. agent-based modeling - Genetic Algorithm (Arifovic, 1994), and
2. representative agent modeling (Granato, Guse and Wong, 2008; Granato, Lo and Wong, 2010)?

We will focus on a simple dynamic supply-demand model (also called the **Cobweb Model**) in this presentation.

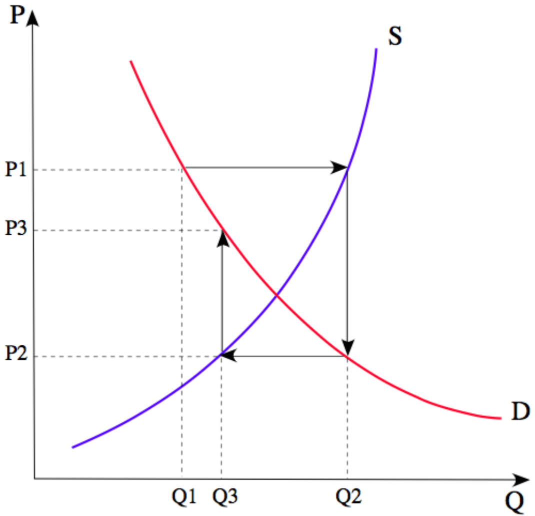
The Cobweb Model - An Introduction

- It is a classic model which illustrates the dynamic process of prices in **agricultural** markets (Kaldor, 1934).
- Due to a lag between planting and harvesting, farmers cannot adjust the amount of agricultural output immediately to fulfill the demand in the market.
- As a result, farmers make their planting decisions today based on the predicted (or forecasted) price of the agricultural product in the next period.
- If farmers expect <u>the price is high</u> in the next period, they would like to <u>plant more</u> today to make more money tomorrow, and vice versa. (The Law of Supply.)

The Cobweb Model - An Introduction

- Assuming that farmers "forecast" the price in the next period based on the price they observe today, that is, $P^e_{t+1} = P_t$.
- If the current price level P_t is high (and is higher than the equilibrium price P^*, which is assumed to be unknown for the farmers). It can be written as: $P_{t=1} > P^*$.
  - **At time $t = 1$ (today),** farmers would like to plant more today for more output ($Q_{t=2}$) sold at a higher price that they expect to happen in the next period. *(All farmers would do the same.)*
  - **At time $t = 2$ (a year later),** all farmers produced more in period 1. Too much output available now. A "surplus" happens in the market (excess supply). Price drops sharply at $t = 2$. The price now goes below the equilibrium: $P_{t=2} < P^* < P_{t=1}$.

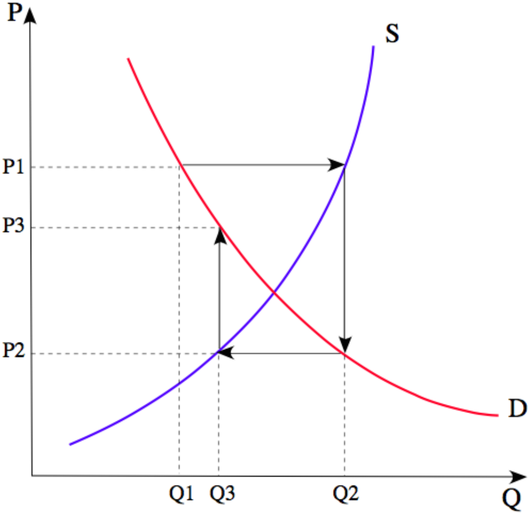
The Cobweb Model - An Introduction



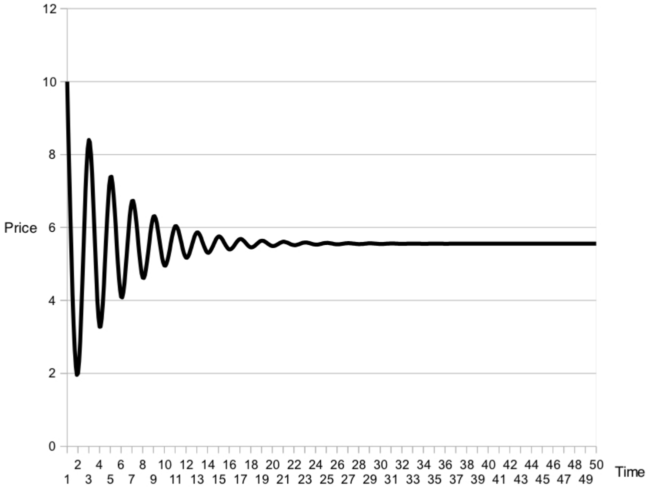
The Cobweb Model - An Introduction

- **Are the farmers happy in period 2? Of course not!**
- **What would the farmers do in period 2?**
  - **When $t = 2$:** Since the today's price is low, they expect the price will also be low at $t = 3$, so they are not motivated to produce and decide to plant less today...
  - **When $t = 3$:** Since all farmers again planted less last period ($t = 2$), the total output level turns out to be very low this time. **Shortage happens!**. ***Price jumps up at $t = 3$!***
- **Woh!! What do you think the farmers would do now?**
  - **Now $t = 3$:** They see the price becomes high again, they are motivated to produce more for the next period ($t = 4$). They decide to plant more today...
- This story keeps going...

The Cobweb Model - Graphical Representation



The Cobweb Model - Price Movements



The Cobweb Model - Representative Agent Hypothesis

- Assuming each firm i chooses a production level q_{it} to maximize its expected profit π_{it}^e.
- The cost function for firm i is:

$$C_{it} = aq_{it} + \frac{1}{2}bmq_{it}^2, \text{ where } a, b > 0.$$

- Given the expected price of the good P_t^e at time t, firm i is maximizing the following profit function:

$$\pi_{it}^e = P_t^e q_{it} - C_{it}(q_{it}) = P_t^e q_{it} - aq_{it} - \frac{1}{2}bmq_{it}^2.$$

- The first order condition for <u>each</u> firm i is:

$$P_t^e - a - bmq_{it} = 0 \Rightarrow q_{it} = \frac{P_t^e - a}{bm}.$$

The Cobweb Model - Representative Agent Hypothesis

- Assuming all firms are identical so that $q_{it} = q_t \; \forall i$, the aggregate supply in the market is:

$$Q_t = \sum_{i=1}^{m} q_{it} = mq_t = \frac{P_t^e - a}{b}, \tag{1}$$

where $m =$ number of firms in the market.

- Assuming that the market demand is a linear function:

$$P_t = \gamma - \theta Q_t, \tag{2}$$

where $Q_t = \sum q_{it}$.

- In equilibrium where (1)=(2), we can derive the following law of motion for the price level:

$$\frac{\gamma - P_t}{\theta} = \frac{P^e - a}{b} \Rightarrow P_t = \frac{\gamma b + a\theta}{b} - \frac{\theta}{b} P_t^e.$$

The Cobweb Theorem

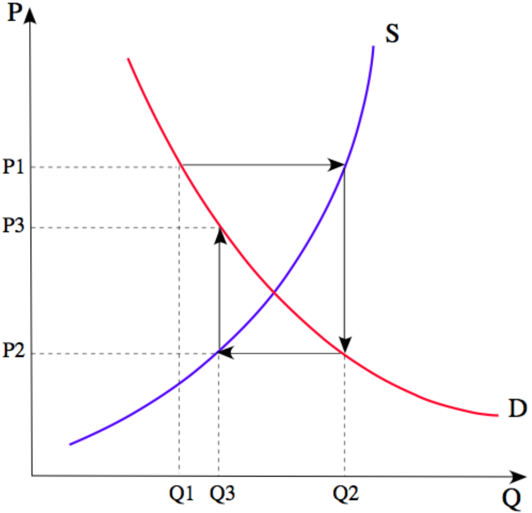
- The dynamics of the price level:

$$P_t = \frac{\gamma b + a\theta}{b} - \frac{\theta}{b} P_t^e. \tag{3}$$

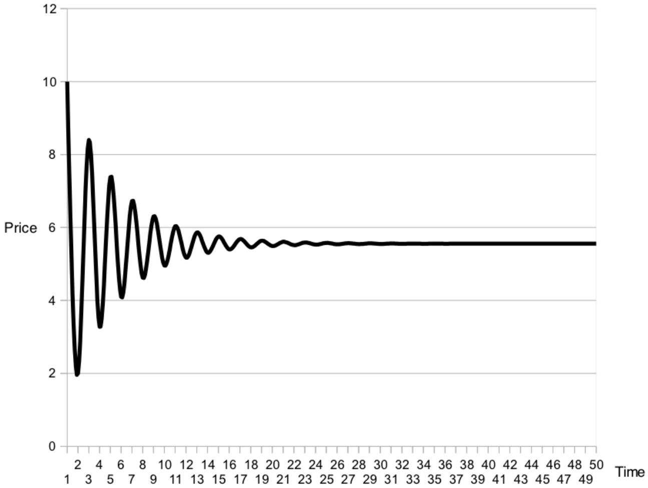
  - Cobweb Theorem:
    - the model is stable (convergent) if $\theta / b < 1$, that is, $\theta < b$.
    - However, the model is unstable (divergent) if $\theta > b$.
- Equation (3) suggests that the expected price can affect the actual price in the economy. According to the agricultural economy, if we expected the price is high (that is, P_t^e is high), then P_t will be low, and vice versa.
- Economists usually write equation (3) as the following reduced form:

$$P_t = \alpha + \beta P_t^e.$$

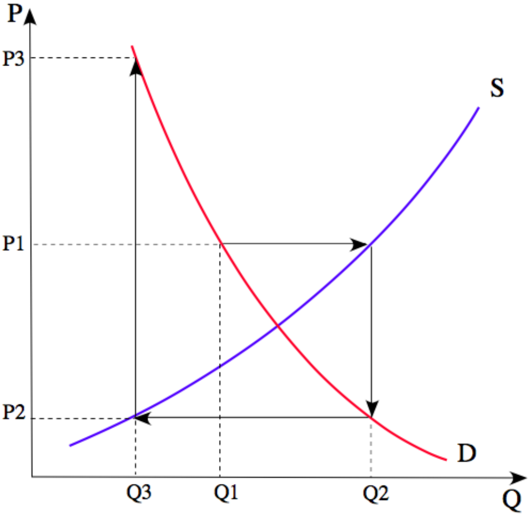
The Cobweb Model - Stable Case



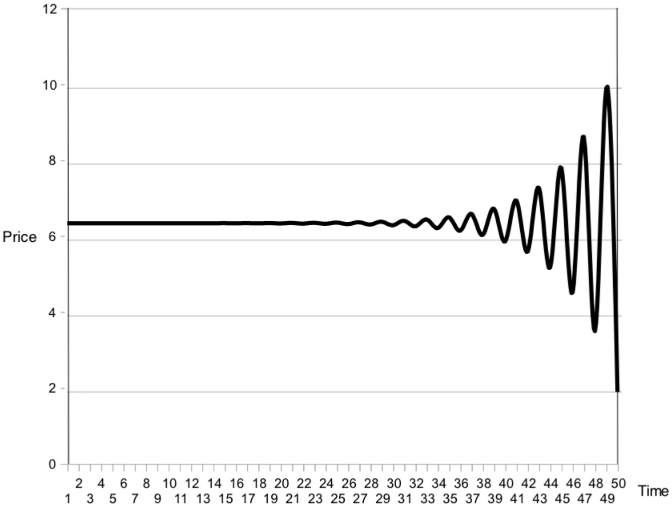
The Cobweb Model - Stable Case



The Cobweb Model - Unstable Case



The Cobweb Model - Unstable Case



Modeling Learning Behavior

What is Genetic Algorithm?

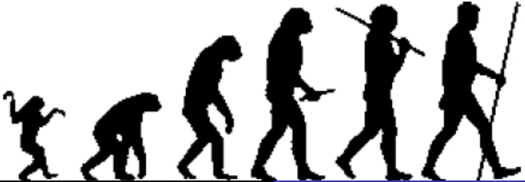
- Does our economy really behave the way we describe in the cobweb model?
- What happens to the Cobweb economy if people/firms behavior differently (based on some alternative assumptions)?
- If we assume all firms are heterogeneous initially and they follow the evolutionary process (genetic algorithm) interacting with each other in the economy, would they end up approaching to the same optimal decision (that is, converging to the same equilibrium)?
- **So what is genetic algorithm?**

Outline

Background

Genetic Algorithm - The Learning Mechanism

- The genetic algorithm (GA), developed by John Holland (1970), is considered one of the evolutionary algorithms inspired by natural evolution with a core concept of "survival of the fittest".
- The GA describes the evolutionary process of a population of genetic individuals with heterogeneous beliefs in response to the rules of nature.



ABM: Genetic Algorithm

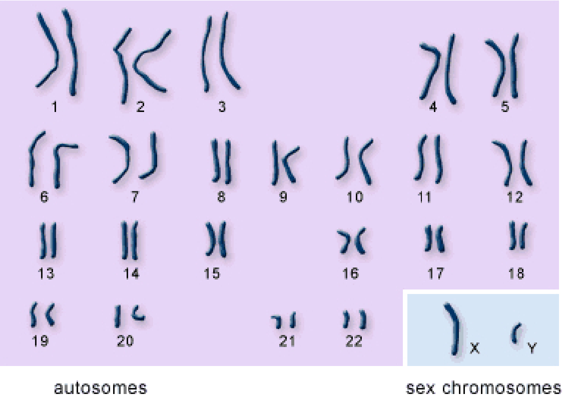
We introduce Arifovic (1994) as an example to investigate if the macro-level stability condition (the cobweb theorem) is necessary for a stable cobweb economy under GA.
We would also like to see how to apply the genetic algorithm on a simple economic model.

Important terms:

- Genes, Chromosomes, and Populations
  - Chromosomes: Genetic individuals making heterogeneous decisions
  - Genes: Elements of a decision that a genetic individual makes
  - Population: A group of genetic individuals with heterogeneous decisions

ABM: Genetic Algorithm

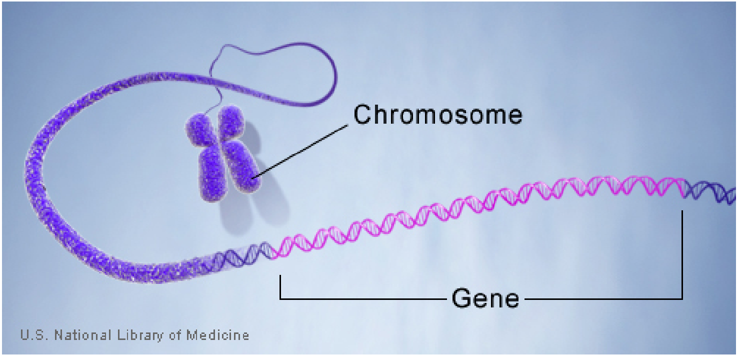
Human Chromosomes - 23 pairs



U.S. National Library of Medicine

ABM: Genetic Algorithm

$$\sum DNA = Gene, \text{ and } \sum Gene = Chromosome$$



ABM: Genetic Algorithm

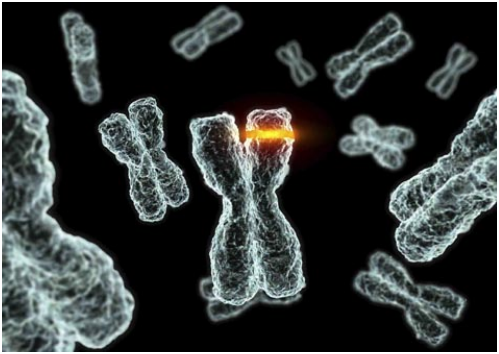
We introduce Arifovic (1994) as an example to investigate if the macro-level stability condition (the cobweb theorem) is necessary for a stable cobweb economy under GA.
We would also like to see how to apply the genetic algorithm on a simple economic model.

Important terms:

- Reproduction, Mutation, and Crossover
  - Reproduction: An individual chromosome is copied from the previous population to a new population.
  - Mutation: One or more gene within an individual chromosome changes value randomly.
  - Crossover: Two randomly drawn chromosomes exchange parts of their genes.

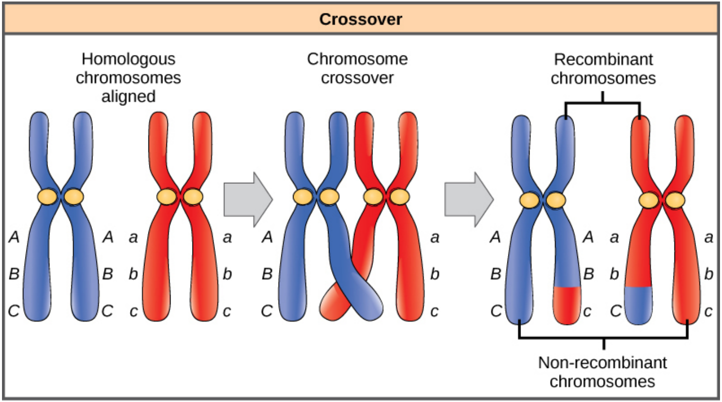
ABM: Genetic Algorithm

Genetic Mutation



ABM: Genetic Algorithm

Genetic Crossover



Computational GA - Genes, Chromosomes, Population

```
C01: 0010100100010101110101010010101010010100101010   Chromosome

C02: 0010100100010101110101010010101010010100101010

C03: 0010100100010101110101010010101010010100101010
C04: 0010100100010101110101010010101010010100101010
C05: 0010100100010101110101010010101010010100101010
C06: 0010100100010101110101010010101010010100101010
C07: 0010100100010101110101010010101010010100101010
C08: 0010100100010101110101010010101010010100101010
C09: 0010100100010101110101010010101010010100101010
C10: 0010100100010101110101010010101010010100101010
C11: 0010100100010101110101010010101010010100101010
C12: 0010100100010101110101010010101010010100101010
C13: 0010100100010101110101010010101010010100101010
C14: 0010100100010101110101010010101010010100101010
```

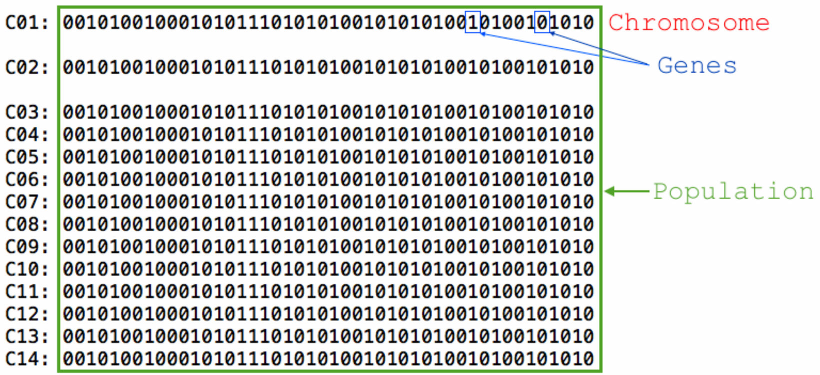
Computational GA - Genes, Chromosomes, Population

```
C01: 0010100100010101110101010010101010010100101010   Chromosome

C02: 0010100100010101110101010010101010010100101010   Genes

C03: 0010100100010101110101010010101010010100101010
C04: 0010100100010101110101010010101010010100101010
C05: 0010100100010101110101010010101010010100101010
C06: 0010100100010101110101010010101010010100101010
C07: 0010100100010101110101010010101010010100101010
C08: 0010100100010101110101010010101010010100101010
C09: 0010100100010101110101010010101010010100101010
C10: 0010100100010101110101010010101010010100101010
C11: 0010100100010101110101010010101010010100101010
C12: 0010100100010101110101010010101010010100101010
C13: 0010100100010101110101010010101010010100101010
C14: 0010100100010101110101010010101010010100101010
```

Computational GA - Genes, Chromosomes, Population



Computational GA - Mutation

The mutation which occurs when one or more gene within an individual chromosome changes value randomly: Agents may change their strategies suddenly through innovations.

```
C01: 0010100100010101110101010010101010010100101010

C01: 001010010 0 01010111010 1 010010101010010100101010

C01: 001010010 1 01010111010 0 010010101010010100101010
```

Computational GA - Crossover

- The crossover which occurs when two randomly drawn chromosomes exchange parts of their genes: *Agents work with others to innovate or develop a new strategy.*

C01: 0010100100010101110101010010101010010100101010

C02: 1010010101001010101001010001000101011110101000

C01: 0010100100010101110101010010 101010010100101010

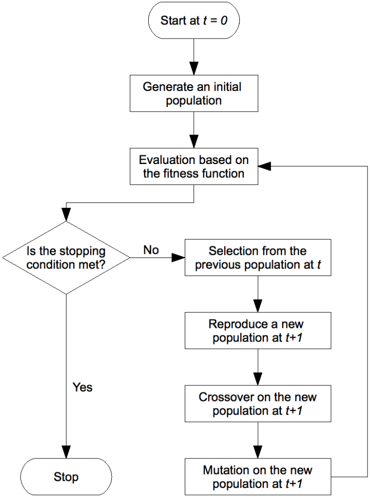
C02: 1010010101001010101001010001 000101011110101000

↓

C01: 0010100100010101110101010010 000101011110101000

C02: 1010010101001010101001010001 101010010100101010

Computational GA - Operational Flowchart



Outline

The Cobweb Theorem & Other Expectations Formations

- Let's revisit the dynamics of the price level:

$$P_t = \frac{\gamma b + a\theta}{b} - \frac{\theta}{b} P_t^e. \tag{4}$$

  - According to Cobweb Theorem, the model is stable (convergent) if $\theta / b < 1$, that is, $\theta < b$. However, the model is unstable (divergent) if $\theta > b$.

- Arifovic discusses three types of expectations formations:
  1. Static expectations (i.e., $P_t^e = P_{t-1}$):
     - The model is stable only if $\theta / b < 1$.
  2. Simple adaptive expectations ($P_t^e = \frac{1}{t}\sum_{s=0}^{t-1} P_s$):
     - The model is stable in both cases (Carlson, 1968).
  3. Least squares learning ($P_t^e = \beta_t P_{t-1}$, β_t = OLS coefficient):
     - The model is stable only if $\theta / b < 1$ (Bray and Savin, 1986).

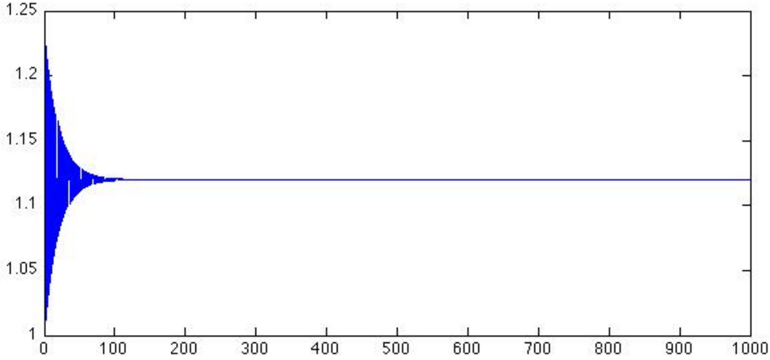
The Cobweb Theorem and Simulation

Parameters	Stable Case $\left(\frac{\theta}{b} < 1\right)$	Unstable Case $\left(\frac{\theta}{b} > 1\right)$
γ	2.184	2.296
θ	0.0152	0.0168
a	0	0
b	0.016	0.016
m	6	6
P^*	1.12	1.12
$Q^*=mq^*$	70	70

Table 12.1: Cobweb Model Parameters

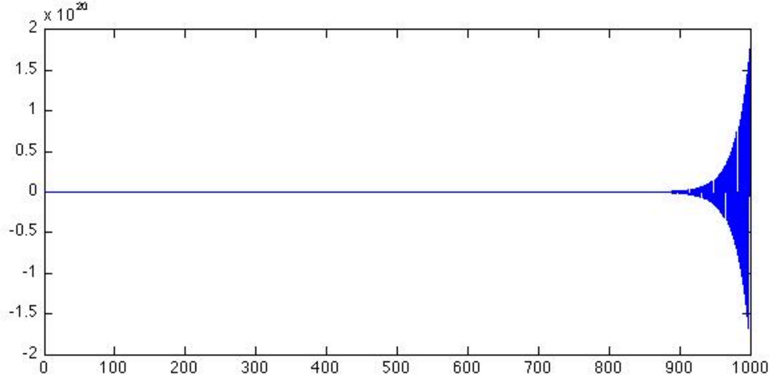
The Cobweb Theorem and Simulation - Static

Static expectations (i.e., $P_t^e = P_{t-1}$): **[Stable Case: $\frac{\theta}{b} < 1$]**



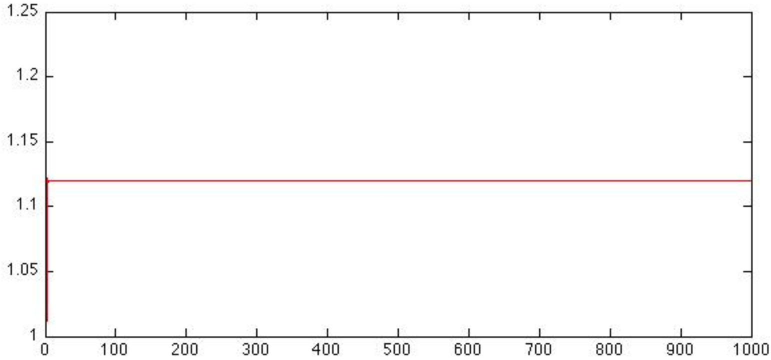
The Cobweb Theorem and Simulation - Static

Static expectations (i.e., $P_t^e = P_{t-1}$): **[Unstable Case: $\frac{\theta}{b} > 1$]**



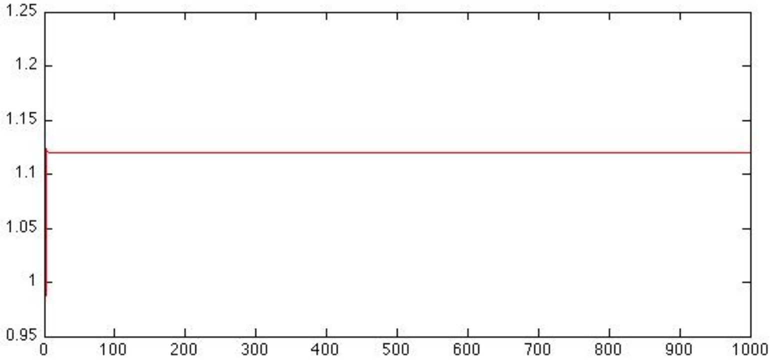
The Cobweb Theorem and Simulation - Adaptive

Simple adaptive expectations ($P_t^e = \frac{1}{t}\sum_{s=0}^{t-1} P_s$): **[Stable Case: $\frac{\theta}{b} < 1$]**



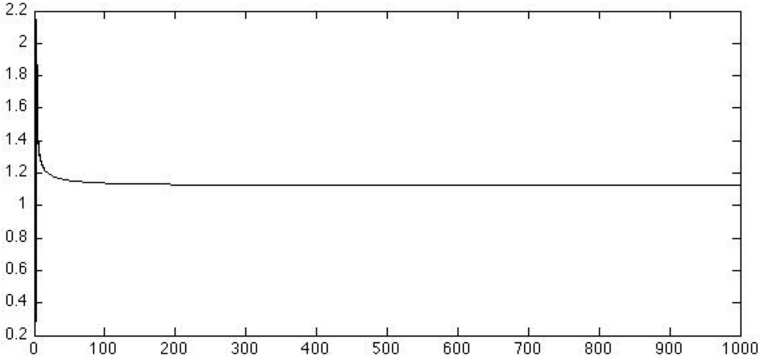
The Cobweb Theorem and Simulation - Adaptive

Simple adaptive expectations ($P_t^e = \frac{1}{t}\sum_{s=0}^{t-1} P_s$): **[Stable Case: $\frac{\theta}{b} > 1$]**



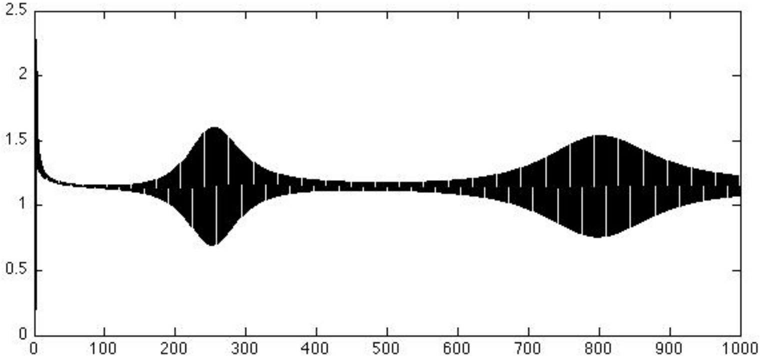
The Cobweb Theorem and Simulation - Least Squares

Least squares learning ($P^e_t = \beta_t P_{t-1}$): **[Stable Case: $\frac{\theta}{b} < 1$]**



The Cobweb Theorem and Simulation - Least Squares

Least squares learning ($P_t^e = \beta_t P_{t-1}$): **[Unstable Case: $\frac{\theta}{b} > 1$]**



The Cobweb Theorem and GA

WHAT ABOUT THE GA LEARNING?

DOES THE COBWEB THEOREM HOLD UNDER THE GA?

Outline

The Basic GA and Arifovic's New GA Operator

- Arifovic (1994) simulates the cobweb model based on three basic genetic operators in the GA simulations:
    - (1) *reproduction*, (2) *mutation*, and (3) *crossover*.
- She also introduces a new operator, called *election*, in the simulations.
- Election is an operator to "examine" the fitness of newly generated (or offspring) chromosomes and then compare them with their parent chromosomes.

New GA Operator - Arifovic (1991, 1994)

- The Rules of Election:
  - Both offspring chromosomes *are elected* to be in the new population at time $t+1$ if $E_t\left(V\left(C_{it+1}^{offspring}\right)\right) > V\left(C_{it}^{Parent}\right)$.
  - However, if only one new chromosome has a higher fitness value than their parents, the one with lower value will not enter the new population, but one of the parents with a higher values stays in the new population.
  - If both new chromosomes have lower values than their parents $E_t\left(V\left(C_{it+1}^{offspring}\right)\right) < V\left(C_{it}^{Parent}\right)$, they cannot enter but their parents stay in the new population.

GA Learning Parameters

Parameters	Stable Case $(\frac{\theta}{b} < 1)$	Unstable Case $(\frac{\theta}{b} > 1)$
γ	2.184	2.296
θ	0.0152	0.0168
a	0	0
b	0.016	0.016
m	6	6
P^*	1.12	1.12
$Q^*{=}mq^*$	70	70

Table 12.1: Cobweb Model Parameters

Set	1	2	3	4	5	6	7	8
Crossover rate: κ	0.6	0.6	0.75	0.75	0.9	0.9	0.3	0.3
Mutation rate: μ	0.0033	0.033	0.0033	0.033	0.0033	0.033	0.0033	0.033

Table 12.2: Crossover and Mutation Rates

GA Learning Parameters

Parameters	Stable Case $(\frac{\theta}{b} < 1)$	Unstable Case $(\frac{\theta}{b} > 1)$
γ	2.184	2.296
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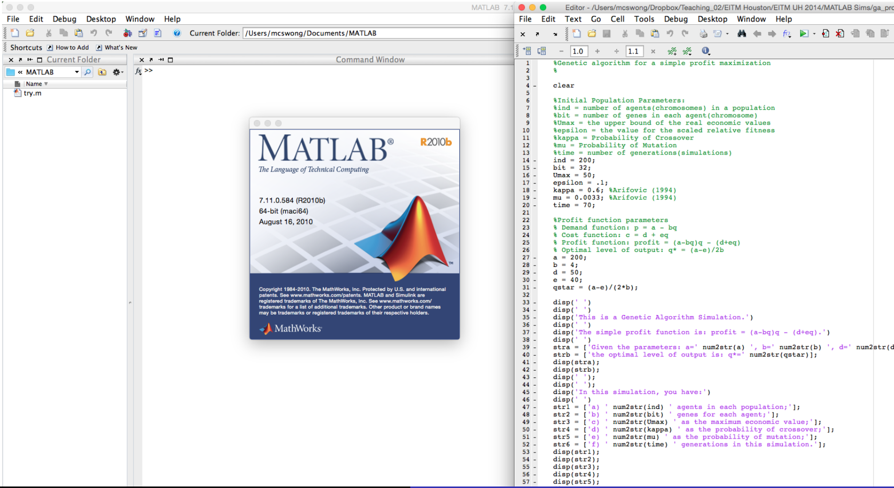
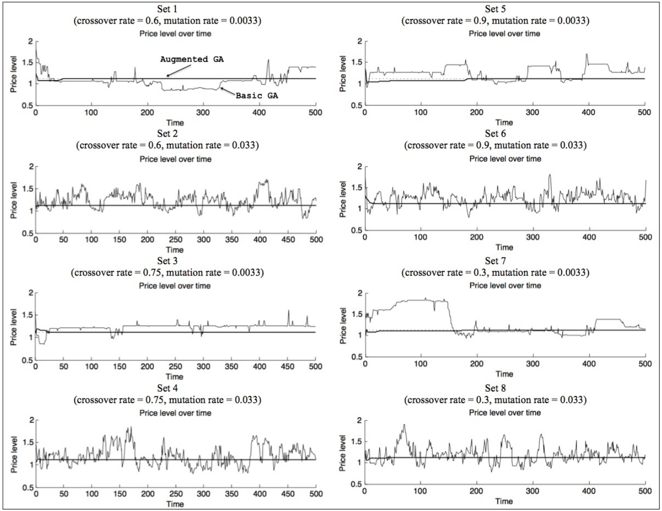
Mutation rate: μ	0.0033	0.033	0.0033	0.033	0.0033	0.033	0.0033	0.033

Table 12.2: Crossover and Mutation Rates

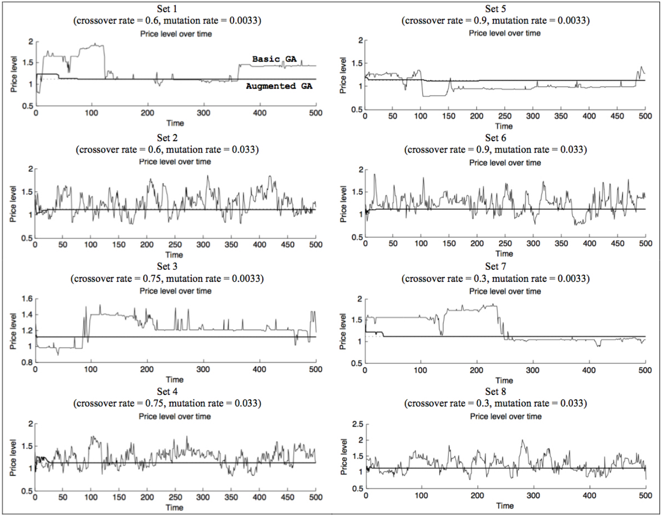
Running Simulations - MATLAB



The GA Simulations - Stable Case ($\theta/b < 1$)



The GA Simulations - Unstable Case ($\theta / b > 1$)



Outline

Conclusions

- Arifovic (1994) introduces the GA procedure as an alternative learning mechanism.
- This alternative learning mechanism mimics social behavior:
  - imitation, communication, experiment, and examination.
- Arifovic uses the GA simulated data to compare with the data generated in human-subject experiments (Wellford, 1989).
  - In an unstable case of the cobweb model, the divergent patterns *do not* happen under both GA learning and human-subject experiments.
  - Price and quantity fluctuate around the equilibrium in *basic* GA learning and human-subject experiments.

Research on Agent-based Modeling

Applications of ABM

- **Poli. Sci.** (Bendor, Diermeier and Ting, APSR 2003; Fowler, JOP 2006)
    - BDT (2003):
        - A computational model by assuming that voters are adaptively rational — voters learn to vote or to stay home in a form of trial-and-error.
        - Voters are reinforced to repeat an action (e.g., vote) in the future given a successful outcome today.
        - The turnout rate is substantially higher than the predictions in rational choice models.
    - Fowler (2006):
        - He revises the BDT model by including habitual voting behavior.
        - Fowler finds his behavioral model is a better fit to the same data BDT use.

Research on Agent-based Modeling

Applications of ABM

- Economics
  - **Econ. Growth** - Beckenbach, et al. (JEE, 2012) - Novelty creating behavior and sectoral growth effects.
  - **Market Structure** - Alemdar and Sirakaya (JEDC, 2003) - Computation of Stackelberg Equilibria.
  - **Policy Making** - Arifovic, Bullard and Kostyshyna (EJ, 2013) - The effects of social learning in a monetary policy context.
    - The Taylor Principle is widely regarded as the necessary condition for stable equilibrium.
    - However, they show that it is not necessary for convergence to REE minimum state variable (MSV) equilibrium under genetic algorithm learning.

Outline

The Complete Cobweb Model

- Recall that the basic Cobweb mode is: $P_t = \frac{\gamma b + a\theta}{b} - \frac{\theta}{b} P_t^e$.
- Assuming that there are other external factors (called w_t) and a stochastic shock (η_t) which can influence the actual price at time t (P_t), we can present the complete Cobweb model as the follows:

$$P_t = \alpha + \beta P_t^e + \gamma w_{t-1} + \eta_t. \tag{5}$$

- Equation (5) has been widely studied in the economic literature since 1960.

An Extended Research Question

What would happen to an economy if there exists information interaction among agents?

The Cobweb Model and Information Diffusion

An Extended Research Question

What would happen to an economy if there exists information interaction among agents?

Answer

The Boomerang Effect: Less-informed agents' forecasts confound those of more-informed agents whenever there is misinterpretation in the information acquisition process. (Granato, Guse and Wong, 2008)

Motivation - Literature Review

1. **Information Diffusion**
   1. Granato and Krause (2000): inflation forecasts of a better educated group influence the less educated group's forecasts, but *not* vice versa.
   2. Financial Economic: Herding behavior (Devenow and Welch, 1996)
2. **Information Asymmetries**
   - Romer and Romer (2000): The Fed has inflation information superior to that of commercial forecasters;
3. **Heterogeneous Information Interpretation**
   - Kandel and Zilberfarb (1999): People do not interpret the existing information in an identical way.

EITM in Practice

Granato, Lo and Wong (2011) is an empirical study (EI) based on the theoretical framework (TM) in Granato, Guse and Wong (2008) where they explore the equilibrium properties in a modified cobweb model with a process of asymmetric information diffusion.

Things to be tested...

- Asymmetric Information Diffusion:
  - Granger Causality Tests
  - Impulse Response of Innovations
- The Boomerang Effect:
  - Rolling-Window Estimations
  - Cointegration estimation
  - Vector Error Correction (VEC) Granger Causality Tests

The Cobweb Model of Inflation Expectations

Using a supply curve and monetary policy, we can derive the reduced-form model of inflation expectations which is consistent with the standard Cobweb model in equation (5):

$$\pi_t = \alpha + \beta E_{t-1}^{*} \pi_t + \gamma w_{t-1} + \eta_t, \tag{6}$$

where $\pi_t = p_t - p_{t-1}$, $\alpha = (1+\theta)^{-1}(\kappa + \bar{m} + \bar{y})$, $\beta = \theta(1+\theta)^{-1}$, $\gamma = (1+\theta)^{-1}(\phi + \lambda)$, and $\eta_t = (1+\theta)^{-1}(\varepsilon_t + \varepsilon_t' + \xi_t)$.

Equation (6) is a self-referential model where inflation depends on:

1. its expectations $\left(E_{t-1}^{*}\pi_t\right)$;
2. exogenous variables (w_{t-1}); and
3. stochastic shocks (η_t)

Inflation Expectations Diffusion Process

The Cobweb model in equation (6) with a process of inflation expectations diffusion:

$$\pi_t = \alpha + \beta E^*_{t-1}\pi_t + \gamma w_{t-1} + \eta_t, \tag{7}$$

New Assumptions (Granato, Guse and Wong, 2008):

1. Information Asymmetry:
   More-informed group (Group H) vs. Less-informed group (Group L)
2. Information Diffusion:
   Group L makes forecasts by observing Group H's forecasts
3. Heterogeneous Information Interpretation:
   Agents in Group L interpret Group H's forecasts differently

Inflation Expectations Diffusion Process

How? Two groups of representative agents: (1) more-informed agents (Group H); and (2) less-informed agents (Group L).

- Group H's Forecasts:

$$E^*_{H,t-1}\pi_t = a_H + b_{1H}x_{t-1} + b_{2H}z_{t-1}, \tag{8}$$

- Group L's Forecasts:

$$E^*_{L,t-1}\pi_t = a_L + b_L x_{t-1} + c_L\hat{\pi}_{t-1}, \tag{9}$$

- where:

$$\hat{\pi}_{t-1} = E^*_{H,t-1}\pi_t + e_{t-1} \ \textit{(Information Diffusion)}, \tag{10}$$

$w_{t-1} = [x_{t-1}, z_{t-1}]'$ *(Asymmetric Information)*, and $e_{t-1} \sim iid\left(0, \sigma_e^2\right)$ is the observation error of observing the expectations of Group H *(Heterogeneous Information Interpretation)*.

Inflation Expectations Diffusion Process

- Assuming that there is a continuum of agents located on the unit interval $[0,1]$ of which a proportion of μ, where $\mu \in [0,1)$ are agents in Group L and $(1-\mu)$ are in Group H.
- Therefore, the expectations of inflation in the market is the weighted average of the expectations from both groups:

$$E^*_{t-1}\pi_t = \mu E^*_{L,t-1}\pi_t + (1-\mu) E^*_{H,t-1}\pi_t \tag{11}$$

Outline

Mixed Expectations Equilibrium (MEE)

The MEE for Group L and Group H:

$$\bar{\varphi}_L = \begin{pmatrix} \bar{a}_L \\ \bar{b}_L \\ \bar{c}_L \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{1-\beta}(1-\bar{c}_L) \\ \frac{\gamma_1}{1-\beta}(1-\bar{c}_L) \\ \frac{\bar{b}_{2H}^2\sigma_z^2}{\bar{b}_{2H}^2\sigma_z^2+(1-\beta\mu)\sigma_e^2} \end{pmatrix} \tag{12}$$

$$\bar{\varphi}_H = \begin{pmatrix} \bar{a}_H \\ \bar{b}_{1H} \\ \bar{b}_{2H} \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{1-\beta} \\ \frac{\gamma_1}{1-\beta} \\ \frac{\gamma_2}{1-\beta+\beta\mu(1-\bar{c}_L)} \end{pmatrix} \tag{13}$$

The Boomerang Effect on Expectations

Group L's observational errors affect Group H's forecast equilibrium:

$$\sigma_e^2 \Rightarrow \bar{b}_{2H}.$$

Forecast Accuracy: Mean Square Errors (MSE)

The Definition of MSE:

$$MSE = E\left(\pi_t - E^*_{t-1}\pi_t\right)^2$$

therefore,

$$MSE_i = E\left(\pi_t - E^*_{i,t-1}\pi_t\right)^2 \text{ for } i \in \{L, H\}$$

The Boomerang Effect on MSE

MSE for Group L and Group H:

$$MSE_L = \left[\frac{\gamma_2 (1-\bar{c}_L)}{1-\beta+\beta(1-\bar{c}_L)\mu}\right]^2 \sigma_z^2 + (1-\beta\mu)^2 \bar{c}_L^2 \sigma_e^2 + \sigma_\eta^2 > \sigma_\eta^2$$

$$MSE_H = (\beta\mu\bar{c}_L)^2 \sigma_e^2 + \sigma_\eta^2 > \sigma_\eta^2$$

The Boomerang Effect on MSE

Group L's observational errors affect Group H's forecast Accuracy: $MSE_H > \sigma_\eta^2$ for $\sigma_e^2 > 0$.

The larger the observation error made by Group L in interpreting Group H's forecasts (larger σ_e^2), the greater the inaccuracy in prediction by Group H (MSE_H).

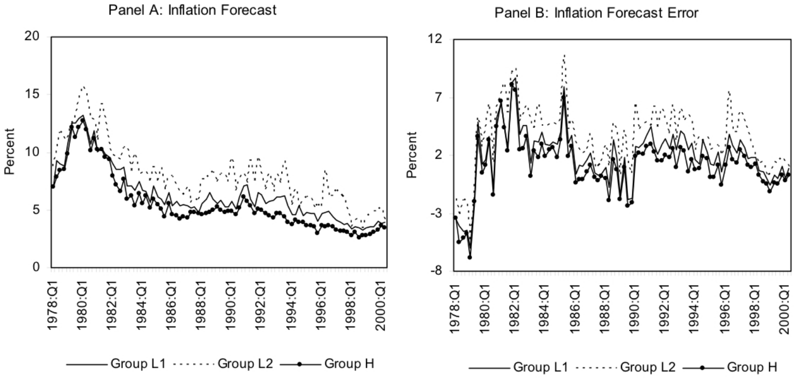
There is a positive relationship between σ_e^2 and MSE_H.

Outline

Data

1. The inflation expectations survey from the Survey Research Center (SRC) at the University of Michigan.
2. Quarterly Data: 1978:Q1 – 2000:Q2.
3. Inflation expectation survey questions:
    - During the next 12 months, do you think that prices in general will up, or go down, or stay where they are now?
    - By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?
4. Data are divided into two groups: Group H and Group L
    - Group H: The respondents with college or graduate degrees
    - Group L: The respondents with less than college degrees
        - L1: High school diploma or some college
        - L2: Less than high school or no high school

Inflation Forecasts and Forecast Errors



Methodology

1. Testing Asymmetric Information Diffusion
    1. Granger Causality Tests
    2. Impulse Response of Innovations
2. Testing the Boomerang Effect
    1. Rolling-Window Estimations of σ_e^2 and MSE_L
    2. Cointegration estimation
    3. Vector Error Correction (VEC) Granger Causality Test

Testing Asymmetric Information Diffusion

Granger Causality Tests

If forecasts of the *higher* educated group Granger-cause those of the *less* educated group?			
Null Hypothesis		Chi-sq statistics [*P*-value]	
a.	Group H does not Granger-cause Group L1	11.401 [0.122]	
b.	Group H does not Granger-cause Group L2	15.522** [0.030]	**Yes, H → L2**
c.	Group L1 does not Granger-cause Group L2	14.253** [0.047]	**Yes, L1 → L2**
If forecasts of the *less* educated group Granger-cause those of the *higher* educated group?			
Null Hypothesis		Chi-sq statistics [*P*-value]	
d.	Group L1 does not Granger-cause Group H	3.897 [0.792]	
e.	Group L2 does not Granger-cause Group H	7.583 [0.371]	
f.	Group L2 does not Granger-cause Group L1	2.603 [0.919]	
** indicates statistical significance at 5 percent.			

Testing Asymmetric Information Diffusion

Impulse Response of Innovations

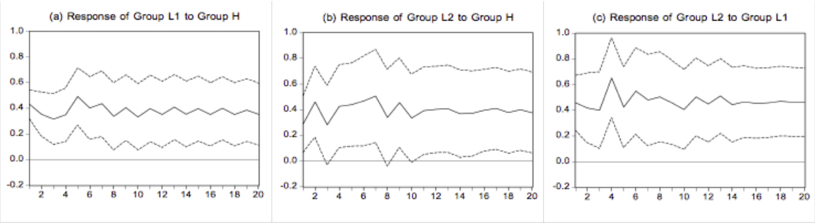


Fig. 2. Accumulated response to one S.D. innovations ± 2 standard errors. Dashed lines denote 95 percent confidence intervals. Group L1 represents agents with a high school diploma or some college. Group L2 represents agents with less than or no high school diploma. Group H represents agents with a college degree or graduate degree. The data source is the SRC at the University of Michigan.

Testing Asymmetric Information Diffusion

Based on Granger causality tests and impulse response of innovations, we find that the less-educated agents observe higher-educated agents in forecasting inflation. There exists the process of asymmetric information diffusion between less- educated and higher-educated groups.

The precondition of the Boomerang Effect is satisfied.

Testing the Boomerang Effect

Testing the Boomerang Effect

1. Rolling-window estimations of σ_e^2 and MSE_H
2. Cointegration estimation
3. Vector error correction Granger causality tests

Step 1a. Rolling-Window Estimation of σ_e^2

Group H's and L's Forecasts:

$$E^*_{H,t-1}\pi_t = a_H + b_{1H}x_{t-1} + b_{2H}z_{t-1}$$
$$E^*_{L,t-1}\pi_t = a_L + b_L x_{t-1} + c_L\hat{\pi}_{t-1}$$

and

$$\hat{\pi}_{t-1} = E^*_{H,t-1}\pi_t + e_{t-1}$$

Testing the Boomerang Effect

Therefore:

$$E_{L,t-1}\pi_t = a_{L,t-1} + b_{L,t-1}x_{t-1} + c_{L,t-1}\left(E_{H,t-1}\pi_t + e_{t-1}\right).$$

We have:

$$e_{t-1} = \frac{E_{L,t-1}\pi_t - a_{L,t-1} - b_{L,t-1}x_{t-1} - c_{L,t-1}E_{H,t-1}\pi_t}{c_{L,t-1}}.$$

We can calculate:

$$\sigma^2_{e_{Lj}} = \frac{\sum_t^{t+s} e^2_{Lj,t-1}}{s-1},$$

for all t and $s =$ size of the rolling window, and $Lj = \{L1, L2\}$.

Testing the Boomerang Effect

Step 1b. Rolling-Window Estimation of MSE_H

By definition, MSE_H can be calculated as:

$$MSE_H = \frac{\sum_t^{t+s} \left(\pi_t - E_{H,t-1}\pi_t\right)^2}{s}$$

Our primary concern is the long run relation between σ_e^2 and MSE_H.

A larger value of σ_e^2 causes MSE_H to increase.

Testing the Boomerang Effect

Steps 2 & 3: Estimation Techniques

1. Unit root test (Dickey-Fuller, 1979; Elliott-Rothenberg-Stock, 1996)
2. Cointegration estimation
3. VEC Grange Causality Tests

Testing the Boomerang Effect

A. Data in levels						
	Augmented Dickey-Fuller Test		Elliott-Rothenberg-Stock Test			
Variable	DF_μ^a	DF_τ^b	Optimal Lag	$DF-GLS_\mu^c$	$DF-GLS_\tau^c$	Conclusion
MSE_H	-2.222	-0.661	3	-0.305	-1.690	I(1)
$\sigma^2_{\tilde{e}_{L1}}$	-0.826	-2.797	3	-0.531	-2.638	I(1)
$\sigma^2_{\tilde{e}_{L2}}$	-1.896	-3.327*	6	-0.743	-3.327**	I(1)
B. Data in first differences						
Variable	DF_μ^a	DF_τ^b	Optimal Lag	$DF-GLS_\mu^c$	$DF-GLS_\tau^c$	Conclusion
MSE_H	-4.536***	-4.966***	2	-2.041*	-2.371**	I(0)
$\sigma^2_{\tilde{e}_{L1}}$	-7.616***	-7.588***	2	-2.957***	-2.973*	I(0)
$\sigma^2_{\tilde{e}_{L2}}$	-7.002***	-6.926***	7	-3.367***	-3.440**	I(0)

***, **, and * indicate statistical significance at 1, 5 and 10 percent, respectively.
a. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. Fuller (1976) 1 and 5 percent critical values for a sample size of 41 equal -3.597 and -2.934, respectively.
b. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. Fuller (1976) 1 and 5 percent critical values for a sample size of 41 equal -4.196 and -3.522, respectively.
c. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. The critical values, not reported here, are calculated from the response surface estimates of Table 1, Cheung and Lai (1995).

Testing the Boomerang Effect

Table 3. Johansen cointegration tests and Granger causality tests in VECM: The (inter-)relation among MSE_H, $\sigma^2_{e,L1}$, and $\sigma^2_{e,L2}$

A: Rank test and cointegrating relation

	Variables in the System							
	$MSE_H, \sigma^2_{e,L1}$ [a] (1)		$MSE_H, \sigma^2_{e,L2}$ [b] (2)		$MSE_H, \sigma^2_{e,L1}, \sigma^2_{e,L2}$ [c] (3)		$\sigma^2_{e,L1}, \sigma^2_{e,L2}$ [d] (4)	
Null Hypothesis	$\hat{\lambda}_{max}$	Trace	$\hat{\lambda}_{max}$	Trace	$\hat{\lambda}_{max}$	Trace	$\hat{\lambda}_{max}$	Trace
No rank	12.82** [11.44]	15.22** [12.53]	8.00 [11.44]	12.20* [12.53]	48.60*** [22.00]	87.52*** [34.91]	6.52 [11.44]	8.80 [12.53]
At most 1 rank	2.40 [3.84]	2.40 [3.84]	4.20 [3.84]	4.20 [3.84]	32.65*** [15.67]	38.92*** [19.96]	2.28 [3.84]	2.28 [3.84]
At most 2 ranks	-	-	-	-	6.27 [9.24]	6.27 [9.24]	-	-
Conclusion	1 cointegrating relation		1 cointegrating relation		2 cointegrating relations		None	
Estimated Cointegration Vector	$(MSE_H, \sigma^2_{e,L1})$ = (1, -29.58)		$(MSE_H, \sigma^2_{e,L2})$ = (1, -21.54)		$(MSE_H, \sigma^2_{e,L1}, \sigma^2_{e,L2})$ = (1, -20.52, -0.79)		None	

Estimated Cointegration Vector	$(MSE_H, \sigma^2_{e,L1})$ = (1, -29.58)	$(MSE_H, \sigma^2_{e,L2})$ = (1, -21.54)	$(MSE_H, \sigma^2_{e,L1}, \sigma^2_{e,L2})$ = (1, -20.52, -0.79)

Testing the Boomerang Effect

B: The direction of causality in VECM

	Variables in the System			
	$MSE_H, \sigma^2_{e,L1}$ [a]	$MSE_H, \sigma^2_{e,L2}$ [b]	$MSE_H, \sigma^2_{e,L1}, \sigma^2_{e,L2}$ [c]	$\sigma^2_{e,L1}, \sigma^2_{e,L2}$ [d]
Null Hypothesis	(1) Chi-sq statistics [*P*-value]	(2) Chi-sq statistics [*P*-value]	(3) Chi-sq statistics [*P*-value]	(4) Chi-sq statistics [*P*-value]
$\sigma^2_{e,L1}$ does not cause MSE_H	14.36*** [0.006]	-	21.04*** [0.007]	-
MSE_H does not cause $\sigma^2_{e,L1}$	3.82 [0.430]	-	5.68 [0.682]	-
$\sigma^2_{e,L2}$ does not cause MSE_H	-	19.43*** [0.000]	30.87*** [0.000]	-
MSE_H does not cause $\sigma^2_{e,L2}$	-	4.72 [0.194]	7.15 [0.521]	-

***, **, and * indicate statistical significance at 1, 5 and 10 percent, respectively. We use the AIC criterion to choose the optimal number of lags to be included in each empirical model. 5 percent critical values, from Osterwald-Lenum (1992), for rank tests are in parentheses.

$\sigma^2_{e,L1}$ does not cause MSE_H	14.36*** [0.006]	-	21.04*** [0.007]
MSE_H does not cause $\sigma^2_{e,L1}$	3.82 [0.430]	-	5.68 [0.682]
$\sigma^2_{e,L2}$ does not cause MSE_H	-	19.43*** [0.000]	30.87*** [0.000]
MSE_H does not cause $\sigma^2_{e,L2}$	-	4.72 [0.194]	7.15 [0.521]

Concluding Remarks

- A larger value of σ_e^2 causes MSE_H to increase.
  - The Boomerang Effect exists in the inflation expectations survey data.
- σ_e^2 granger causes MSE_H, but not vice versa.
  - The finding of Boomerang Effect is robust using the inflation expectations survey data.

Concluding Remarks

Thank You.

Questions?

Extra 1: Profit Maximization Problem with GA

How do we perform the genetic algorithm simulation?

Profit Maximization

1. Profit function: $\pi = p \times q - c(q)$.
2. Demand: $p = a - bq$.
3. Supply (cost function): $c = d + eq$.
4. Maximizing profit: $\max_q \pi = (a - bq)\, q - (d + eq)$.
5. Optimal level of output: $q^* = (a - e)/2b$.

How to do GA?

```
Generating Initial Population

001001001000110101101010
101101010001001001000110
001001001000110101101010
010010010001101011010100
001001001000110101101010
101100100100011001010001
001001001000110101101010
101101010001001001000110
001001001000110101101010
101101010001001001000110
001001001000110101101010
101101010000100010010110
```

How to do GA?

```
Evaluation based on the Fitness Function

001001001000110101101010
101101010001001001000110
001001001000110101101010
010010010001101011010100
001001001000110101101010
101100100100011001010001
001001001000110101101010
101101010001001001000110
001001001000110101101010
101101010001001001000110
001001001000110101101010
101101010000100010010110
```

How to do GA?

```
Reproduction according to the fitness value

001001001000110101101010 (reproduced with 10% probability)
101101010001001001000110 (reproduced with 5% probability)
001001001000110101101010 (reproduced with 20% probability)
010010010001101011010100 (reproduced with 0% probability)
001001001000110101101010
101100100100011001010001
001001001000110101101010
101101010001001001000110
001001001000110101101010
101101010001001001000110
001001001000110101101010
101101010000100010010110
```

How to do GA?

```
After Reproduction, Genetic Operations
1. mutation
2. crossover

001001001000110101101010
101101010001001001000110
001001001000110101101010
010010010001101011010100
001001001000110101101010
101101010001001001000110
101101010001001001000110
101101010001001001000110
001001001000110101101010
101101010001001001000110
001001001000110101101010
101101010000100010010110
```

Notations under the GA

- Chromosome C_i consists of a set of 0 and 1, where L is the length of a chromosome (the number of genes).
- $B^{max}(C_i) = 2^L - 1$ represents the maximum numerical value of a chromosome with the length L.
    - For example, if $L = 10$, the maximum value of a chromosome:

$$B(1111111111) = 2^{10} - 1 = 1023.$$

- We can use the B operator to compute a numerical value of a chromosome (e.g., $C_i = 0100101110$) :

$$\begin{aligned} B(0100101110) &= 0 \times 2^9 + 1 \times 2^8 + 0 \times 2^7 + 0 \times 2^6 + \\ & 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + \\ & 1 \times 2^1 + 0 \times 2^0 = 302. \end{aligned}$$

Notations under the GA

- Assume that there are $M = 8$ genetic individuals. For $L = 10$, we can generate an initial genetic population P_0 in an $M \times L$ matrix (that is, 8×10 matrix):
- For example:

$$P_0 = \begin{matrix} 0100101110 \\ 1110101010 \\ 0101110100 \\ 0100001010 \\ 1110101000 \\ 0101101101 \\ 1100101010 \\ 0100011100 \end{matrix}$$

Notations under the GA

- According to the problem of profit maximization, if $a = 200$, $b = 4$, and $e = 40$, then $q^* = 20$.
- In this case, the maximum value of a chromosome can be too large for this problem $(B^{max} = 1023)$.
- We can define a maximum economic value for a chromosomes $V(C_i)$ based on the following value function:

$$V(C_i) = \frac{U^{max}}{B^{max}} \times B(C_i),$$

where $V(C_i) \in [0, U^{max}]$ for $B(C_i) \in [0, B^{max}]$, and U^{max} is the maximum economic value in the problem.

Notations under the GA

- An economic value for a chromosomes $V(C_i)$ based on the following value function:

$$V(C_i) = U^{max} \times \frac{B(C_i)}{B^{max}}.$$

- For example, given the maximum output level is $U^{max} = 100$, and $C_i = 0100101110$ (i.e., $B(C_i) = 302$), we can calculate the output level for firm i:

$$q_i = V(C_i) = 100 \times \frac{302}{1023} = 29.52 \approx 30.$$

Notations under the GA

- Is firm i doing a good job? We need to evaluate firm i using a fitness function $F(C_i)$.
- The profit function is used as the fitness function in this case:

$$
\begin{aligned}
F(C_i) &= \pi(V(C_i)) \\
&= \pi(q_i) = (a - bq_i)\, q_i - (d + eq_i).
\end{aligned}
$$

- In this case,

$$
\begin{aligned}
F(C_i) &= \pi(V(C_i)) \\
&= \pi(29.52) = (200 - 4(29.52))(29.52) - (50 + 40(29.52)) \\
&= 1187.48.
\end{aligned}
$$

- The maximum profit is (for $q* = 20$):

$$
F^{max} = \pi(q^*) = \pi(20) = 1550.
$$

The GA Operators

Reproduction $\Rightarrow$ Evolutionary Dynamics

- Reproduction is a genetic operator where an individual chromosome is copied from the previous population to a new population.
- The probability of being drawn for each chromosome is calculated based on the fitness value.
  - Higher fitness value $\Rightarrow$ higher probability of being drawn to the new population.
- The relative fitness function is:

$$R(C_{i,t}) = \frac{F(C_{i,t})}{\sum_{m=1}^{M} F(C_{m,t})},$$

  where $\sum_{i \in M} R(C_{i,t}) = 1$.
- The relative fitness value $R(C_{i,t})$ gives us the probability chromosome i is copied to the new population at time $t+1$.

The GA Operators
Reproduction

- What if $F(C_{i,t})$ is negative for some firm i? (a negative profit?)
- Goldberg (1989) proposes a scaled relative fitness function:

$$S(C_{i,t}) = \frac{F(C_{i,t}) + A}{\sum_{m=1}^{M} [F(C_{m,t}) + A]} = \frac{F(C_{i,t}) + A}{\sum_{m=1}^{M} F(C_{m,t}) + MA},$$

where A is a constant such that $A > -\min_{C_i \in P_t} F(C_{i,t})$.

The GA Operators

Crossover

- A crossover point will be randomly chosen to separate each chromosome into two sub-strings.
- Two "offspring" chromosomes will be formed by swapping the right-sided parents' substrings with probability κ.

```
C01: 0010100100010101110101010010101010010100101010

C02: 1010010101001010101001010001000101011110101000


C01: 00101001000101011101010100010 10101001010010101010

C02: 10100101010010101010010100001 0001010111110101000

C01: 00101001000101011101010100010 0001010111110101000

C02: 10100101010010101010010100001 10101001010010101010
```

The GA Operators

Crossover

Assuming that there are $M = 6$ individuals in the population (each chromosome has 20 genes) :

```
[6x20] matrix
C01: 10010100100110101010
C02: 10101010010001101100
C03: 01101100101000110110
C04: 11011001010001110100
C05: 10110010111101100101
C06: 10110101111011001010
```

The GA Operators

Crossover

Therefore, there are $20 - 1 = 19$ possible positions for crossover. We randomly pick a position for each pair of chromosomes.

```
Break the population into 3 groups.
Randomly pick a position between Position 1 and Position 19

C01: 10010100100110101010
C02: 10101010010001101100

C03: 01101100101000110110
C04: 11011001010001110100

C05: 10110010111101100101
C06: 10110101111011001010
________________________________________
```

The GA Operators

Crossover

Given $\kappa = 0.3$, the position for the 1st pair is 8, the 2nd pair is 3, and the 3rd is 0.

```
C01: 100101001001_10101010 [Position 8]
C02: 101010100100_01101100

C03: 01101100101000110_110 [Position 3]
C04: 11011001010001110_100

C05: 10110010111101100101  [Position 0]
C06: 10110101111011001010
______________________________________
```

The GA Operators
Crossover

This is a new population after crossover.

```
C01: 100101001001_01101100 [Position 8]
C02: 101010100100_10101010

C03: 01101100101000110_100 [Position 3]
C04: 11011001010001110_110

C05: 10110010111101100101_ [Position 0] – NO CROSSOVER
C06: 10110101111011001010_
_______________________________________
```

The GA Operators

Mutation

- Every gene within a chromosome has a small probability, μ, changing in value, independent of other positions.

```
C01: 0010100100010101110101010010101010010100101010
```

$$\texttt{C01: 001010010 } \boxed{\texttt{0}} \texttt{ 01010111010 } \boxed{\texttt{1}} \texttt{ 0100101010100101001010}$$
$$\downarrow \qquad\qquad \downarrow$$
$$\texttt{C01: 001010010 } \boxed{\texttt{1}} \texttt{ 01010111010 } \boxed{\texttt{0}} \texttt{ 0100101010100101001010}$$

Defining Parameter Values

```
%Initial Population Parameters:
%ind = number of agents(chromosomes) in a population
%bit = number of genes in each agent(chromosome)
%Umax = the upper bound of the real economic values
%epsilon = the value for the scaled relative fitness
%kappa = Probability of Crossover
%mu = Probability of Mutation
%time = number of generations(simulations)
ind = 200;
bit = 32;
Umax = 50;
epsilon = .1;
kappa = 0.6; %Arifovic (1994)
mu = 0.0033; %Arifovic (1994)
time = 500;

%Profit function parameters
% Demand function: p = a - bq
% Cost function: c = d + eq
% Profit function: profit = (a-bq)q - (d+eq)
% Optimal level of output: q* = (a-e)/2b
a = 200;
b = 4;
d = 50;
e = 40;
qstar = (a-e)/(2*b);
```

Creating an Initial Population

```
%Value Function and Definitions
Bmax = (2 .^ bit) - 1;
m = ind;
n = bit;

%Generate the Initial Population: "gen"
gen = rand(m,n);
for i=1:m
    for j=1:n
        if gen(i,j)<.5;
            gen(i,j)=0;
        else
            gen(i,j)=1;
    end
end
end
```

Converting Binary Value into Numerical Value

```
%Calculate the real value of each chromosome: "BC"
m2 = 2 * ones(n,1);
for i=1:n
    m2(i,1)=m2(i,1).^(n-i);
end

BC = ones(m,1);
for i=1:m
    BC(i,1)=gen(i,:) * m2; %Converting Binary # to Decimal # for each i
end
```

- For example,

$$\begin{aligned} B(0100101110) &= 0\times 2^9 + 1\times 2^8 + 0\times 2^7 + 0\times 2^6 + \\ &\quad 1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + \\ &\quad 1\times 2^1 + 0\times 2^0 = 302. \end{aligned}$$

Notations under the GA

- An economic value for a chromosomes $V(C_i)$ based on the following value function:

$$V(C_i) = \frac{U^{max}}{B^{max}} \times B(C_i).$$

- For example, given the maximum output level is $U^{max} = 100$, and $C_i = 0100101110$ (i.e., $B(C_i) = 302$), we can calculate the output level for firm i:

$$q_i = V(C_i) = \frac{100}{1023} \times 302 = 29.52 \approx 30.$$

Notations under the GA

- Is firm i doing a good job? We need to evaluate firm i using a fitness function $F(C_i)$.
- The profit function is used as the fitness function in this case:

$$\begin{aligned} F(C_i) &= \pi(V(C_i)) \\ &= \pi(q_i) = (a - bq_i)\, q_i - (d + eq_i). \end{aligned}$$

- In this case,

$$\begin{aligned} F(C_i) &= \pi(V(C_i)) \\ &= \pi(29.52) = (200 - 4(29.52))(29.52) - (50 + 40(29.52)) \\ &= 1187.48. \end{aligned}$$

- The maximum profit is (for $q* = 20$):

$$F^{max} = \pi(q^*) = \pi(20) = 1550.$$

The GA Operators

Reproduction ⇒ Evolutionary Dynamics

- Reproduction is a genetic operator where an individual chromosome is copied from the previous population to a new population.
- The probability of being drawn for each chromosome is calculated based on the fitness value.
  - Higher fitness value ⇒ higher probability of being drawn to the new population.
- The relative fitness function is:

$$R(C_{i,t}) = \frac{F(C_{i,t})}{\sum_{m=1}^{M} F(C_{m,t})},$$

  where $\sum_{i \in M} R(C_{i,t}) = 1$.
- The relative fitness value $R(C_{i,t})$ gives us the probability chromosome i is copied to the new population at time $t+1$.

The GA Operators

Reproduction

```
%This is the code for Reproduction
norm_fit = SC
selected = rand(size(SC))
sum_fit = 0;
for i=1:length(SC)
    sum_fit = sum_fit + norm_fit(i)
    index = find(selected<sum_fit)
    selected(index) = i*ones(size(index))
end
gen = gen(selected,:)
```

Goldberg (1989) proposes a scaled relative fitness function:

$$S\left(C_{i,t}\right) = \frac{F\left(C_{i,t}\right) + A}{\sum_{m=1}^{M}\left[F\left(C_{m,t}\right) + A\right]} = \frac{F\left(C_{i,t}\right) + A}{\sum_{m=1}^{M} F\left(C_{m,t}\right) + MA},$$

where A is a constant such that $A > -\min_{C_i \in P_t} F\left(C_{i,t}\right)$.

The GA Operators

Reproduction

```
>> norm_fit = SC

norm_fit =

    0.1283
    0.1230
    0.1182
    0.0000
    0.1276
    0.0785
    0.0780
    0.1271
    0.0927
    0.1266

>>   selected

selected =

    10
     1
     3
     5
     1
     1
     1
    10
    10
     5
```

The GA Operators

Crossover

```
%This is the code for Crossover (Point & Pairwise)
%size(gen,1) = ind = number of individual
%size(gen,2) = bit = number of genes
sites = ceil(rand(size(gen,1)/2,1)*(size(gen,2)-1))
sites = sites.*(rand(size(sites))<kappa)
for i = 1:length(sites)
    newgen([2*i-1 2*i],:) = [gen([2*i-1 2*i],1:sites(i)) ...
                             gen([2*i 2*i-1],sites(i)+1:size(gen,2))]
end
gen=newgen
```

The GA Operators

Crossover

```
>> rand(size(gen,1)/2,1)

ans =

    0.6378
    0.3878
    0.8372
    0.7663
    0.1256

>> size(gen,2)-1

ans =

    31

>> ceil(rand(size(gen,1)/2,1)*(size(gen,2)-1))

ans =

     3
    21
    12
     4
    20
```

The GA Operators

Mutation

```
%This is the code for Mutation
mutated = find(rand(size(gen))<mu)
newgen = gen
newgen(mutated) = 1-gen(mutated)
gen=newgen;
ngen=newgen;
```

The GA Operators
Mutation

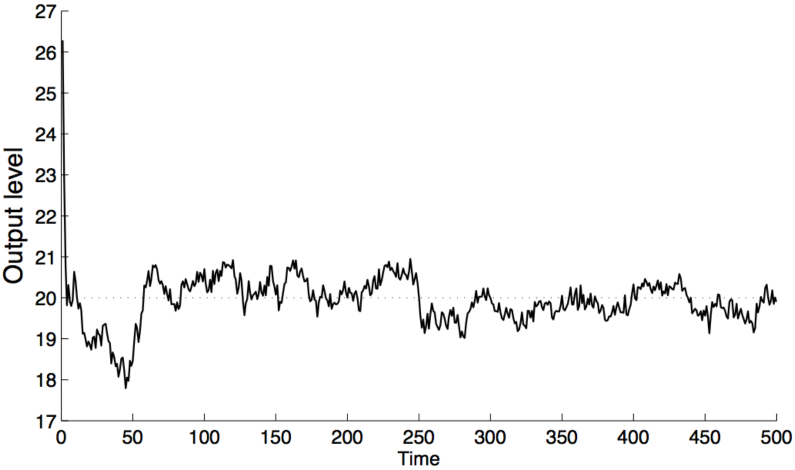
```
mutated =

     3
    43


newgen =

     1     0     1     1     1     1     1     0
     1     1     0     0     1     0     0     1
     0     1     1     1     0     0     1     1
     0     1     1     1     0     0     1     1
     0     1     0     0     0     1     0     1
     0     0     0     1     1     1     0     1
     0     1     1     1     0     0     1     1
     0     1     1     1     0     0     1     1
     0     1     0     0     0     1     0     1
     1     0     0     0     0     0     0     0


newgen =

     1     0     1     1     1     1     1     0
     1     1     0     0     1     0     0     1
     1     1     1     1     1     0     1     1
     0     1     1     1     0     0     1     1
     0     1     0     0     0     1     0     1
     0     0     0     1     1     1     0     1
     0     1     1     1     0     0     1     1
     0     1     1     1     0     0     1     1
     0     1     0     0     0     1     0     1
     1     0     0     0     0     0     0     0
```

The Basic GA Simulations

- Market Parameters:
  - Demand: $a = 200$, and $b = 400$.
  - Supply: $d = 50$, and $e = 40$.
  - Optimal output: $q^* = 20$.
- GA Parameters:
  - $M = 200$ (200 genetic agents)
  - $L = 16$, therefore $B^{max} = 65535$.
  - $U^{max} = 50$ (maximum output $q^{max} = 50$)
  - $\kappa = 0.3$ (probability of crossover)
  - $\mu = 0.0033$ (probability of mutation)
  - $t = 500$ (500 generations)

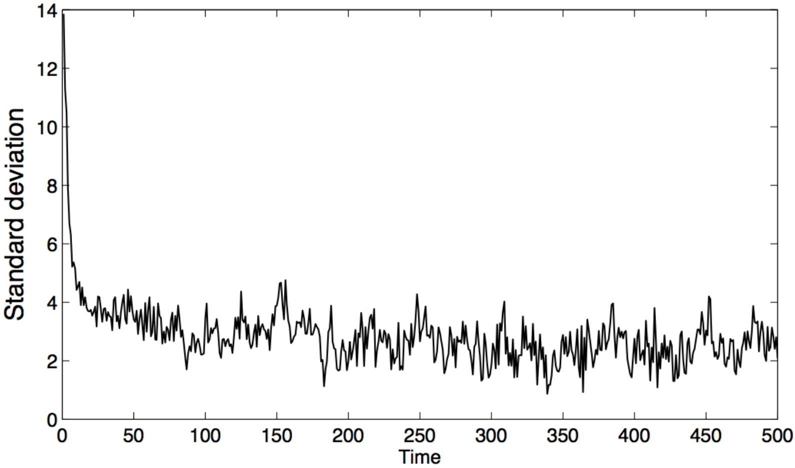
The Basic GA Simulations

The Output Level over time

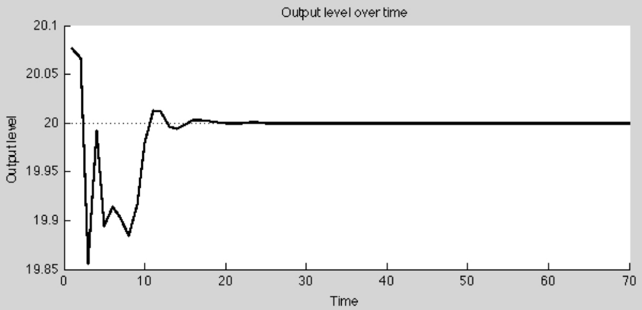


The Basic GA Simulations

The Standard Deviation of Output Level over time

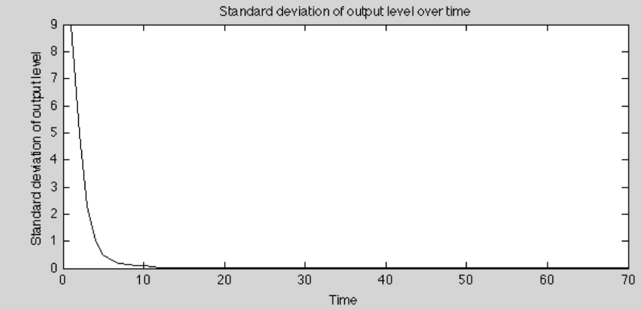


The Augmented GA Simulations
The Output Level over time



The Augmented GA Simulations

The Standard Deviation of Output Level over time

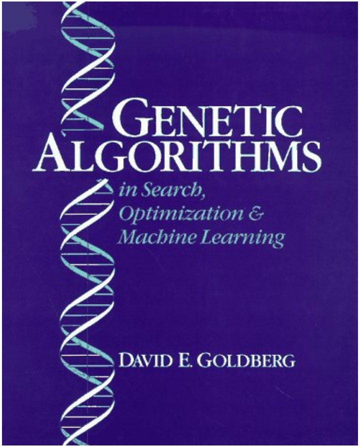


Concluding Remarks

- Why do we use the GA (or ABM in general) for political science / economics research??
  - Some models are mathematically intractable (we cannot find a closed-form equilibrium).
  - No strong assumptions imposed (such as, efficient markets, rational agents, representative agent hypothesis).
  - It allows non-linearity in a theoretical model.
  - It is relatively easier to capture equilibrium (equilibria) in a multi-national, multi-sector model.

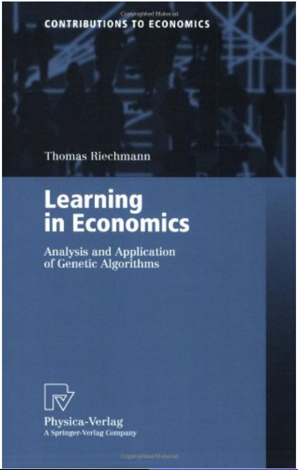
Learn GA Learning?

Genetic Algorithms in Search, Optimization, and Machine Learning (David E. Goldberg, 1989)



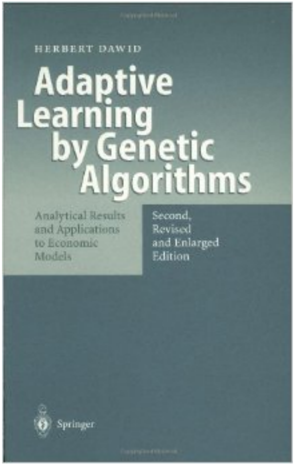
Learn GA Learning?

Learning in Economics: Analysis and Application of Genetic Algorithms (Thomas Riechmann, 2001)



Learn GA Learning?

Adaptive Learning by Genetic Algorithms: Analytical Results and Applications to Economic Models (Herbert David, 2012)



Extra 2: The Setup of Inflation Expectations Model

The Lucas Supply Curve is:

$$y_t = \bar{y} + \theta \left(p_t - E^*_{t-1} p_t \right) + \varepsilon_t \tag{14}$$

and QTM is:

$$m_t + v_t = p_t + y_t, \tag{15}$$

where velocity depends on some exogenous observables (w_{t-1}):

$$v_t = \kappa + \lambda w_{t-1} + \varepsilon'_t \tag{16}$$

and money supply (m_t) is represented as a policy rule:

$$m_t = \bar{m} + \phi w_{t-1} + \xi_t, \tag{17}$$

where $\bar{m}$ is a constant money stock.

Concluding Remarks

Thank You.

Questions?

Sources of Figures

- Evolutionary figure: http://mme.uwaterloo.ca/~fslien/ga/ga.html
- Human chromosome: http://ghr.nlm.nih.gov/handbook/illustrations/chromosomes.jpg
- Genetic mutation: http://farm3.static.flickr.com/2350/1583336323_33661151a2_o.jpg
- Genetic crossover: http://cnx.org/content/m45471/latest/Figure_08_03_06.jpg